

CLASSIFICATION AND DESIGN OF ISLAMIC GEOMETRIC PATTERNS USING COMPUTER GRAPHICS

Ahmad M. Aljamali and M Roberts Masillamani

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Abstract

This paper presents an innovative method to classify and design Islamic geometric patterns (IGP) using computer software. The researcher traced the existing systems associated with the classification of Islamic geometric patterns i.e. 7-frieze patterns [Gla99] and 17-wallpaper patterns theories [Lee98]. Also researcher traced the conventional design methodologies that are based on principles of classical gridding systems, which are hierarchical in nature. The researcher found out to distinguish between classification of a pattern (collection of unit patterns) and classification of a design (collection of grids and geometric attributes). Classification of a pattern involves repeating the unit pattern by isometry formulas (translation, mirroring, rotation and glide reflections) to generate a pattern that can be classified as 7-freize patterns or the 17-wallpaper patterns. Classification of a design involves the normalisation of the grids and geometries. In this paper the researcher presented an argument those pattern theories are purely base models. A refined classification with specific focus on the girding system of the star/rosette of IGP was required. The task of presenting a new method of classification rightly demanded a geometrically and scientifically validated method. Researcher has been successful in developing such a method and presenting it in user-friendly software. Researcher generated software that draws the grids of any particular IGP star/rosette design and displays its classification instantly. This approach resulted in a measurable method of classification for any given Islamic geometric design. The software is facilitated to identify the IGP star/rosette sub-motif grid from its gridding system of classification which will allow the user to explore and design the sub-motif pattern, motif pattern, unit pattern and finally the pattern in x-y direction.

Keywords: Classification, Design, IGP, Pattern Theories, Normalisation, Motif, Software.

1. INTRODUCTION

Geometric patterns are popularly associated with Islamic art, largely due to their iconic quality. These abstract designs not only adorn the surfaces of

monumental Islamic architecture but also function as the major decorative element on a vast array of objects of all types. Islamic artists appropriated key elements from the classical tradition, and later complicated and elaborated them in order to invent a new form of decoration that intense the importance of unity and order. The works of Euclid and Pythagoras were among the earliest to be translated into Arabic. The study of geometry also fed an ardent preoccupation with the stars and astronomy. Driven by the religious passion for abstraction and the related doctrine of unity -- al-tawhid, the Muslim intellectuals recognised the importance of geometry as the unifying intermediary between the material and the spiritual world. The significant intellectual contributions of Islamic mathematicians, astronomers, and scientists were essential for the creation of this unique new style. Theorists have often presented a long-standing historic awe to group geometric patterns in either 7-frieze patterns or 17-wallpaper patterns like Lee [Lee98], Grunbaum [Gru86], Glassner [Gla99], and Schattschneider [Sch89] to get a holistic view of the entire spectrum of work on this subject. This paper presents an argument that existing symmetrical classification methods like the 7-frieze patterns and the 17-wallpaper patterns are purely base models for pattern classification and not to be considered as the classification of Islamic geometric unit designs. Based on the geometries and the grids used for generating the IGP star/rosette, a computer software was developed for Classification Study and Design.

2. CLASSIFICATIONS

One type of pattern classification is the 7 frieze patterns. A frieze pattern is defined as mathematical concept to classify designs on two-dimensional surfaces which are repetitive in one direction, based on the symmetries in the pattern. It is also refers that such patterns occur frequently in architecture and decorative art. There are seven different frieze patterns. Any transformation of the plane leaving this pattern invariant can be decomposed into a translation, $(x,y) \rightarrow (n+x,y)$, optionally followed by a reflection in either the horizontal axis, $(x,y) \rightarrow (x,-y)$, or the vertical axis, $(x,y) \rightarrow (-x,y)$, provided that this axis is chosen through or midway between two dots, or a rotation by 180°, $(x,y) \rightarrow (-x,-y)$ (ditto).

The other type of pattern classification is the 17 wallpaper patterns. Arthur Cayley (1821-1895) and James Sylvester (1814-1897) were the earliest mathematicians to write about these transformations. They suggested that the sets of transformations can be used to classify geometries. In 1872, Felix Klein classified geometries by applying the definitions. Of particular importance is the application of transformations to crystallography which led to the lattice theory and the famous 17 wallpaper patterns. George Pólya in 1924 was the first one to prove that there were only 17 possible patterns. Each wallpaper tiling is labelled with the name used by the International Union of Crystallography since 1952. The wallpaper pattern theories present arrangement of patterns as the benchmarks, which enable us to determine the type of pattern used to arrange the design rather than classifying the unit pattern. In figures 1 and 2 we demonstrate that the 7-frieze pattern [Gla99] and the 17-wallpaper patterns [Lee] theories misdirecting the

classification of Islamic geometric patterns.

3. DESIGNS

Different researchers [Mul44] [Bel56] [Cri76]

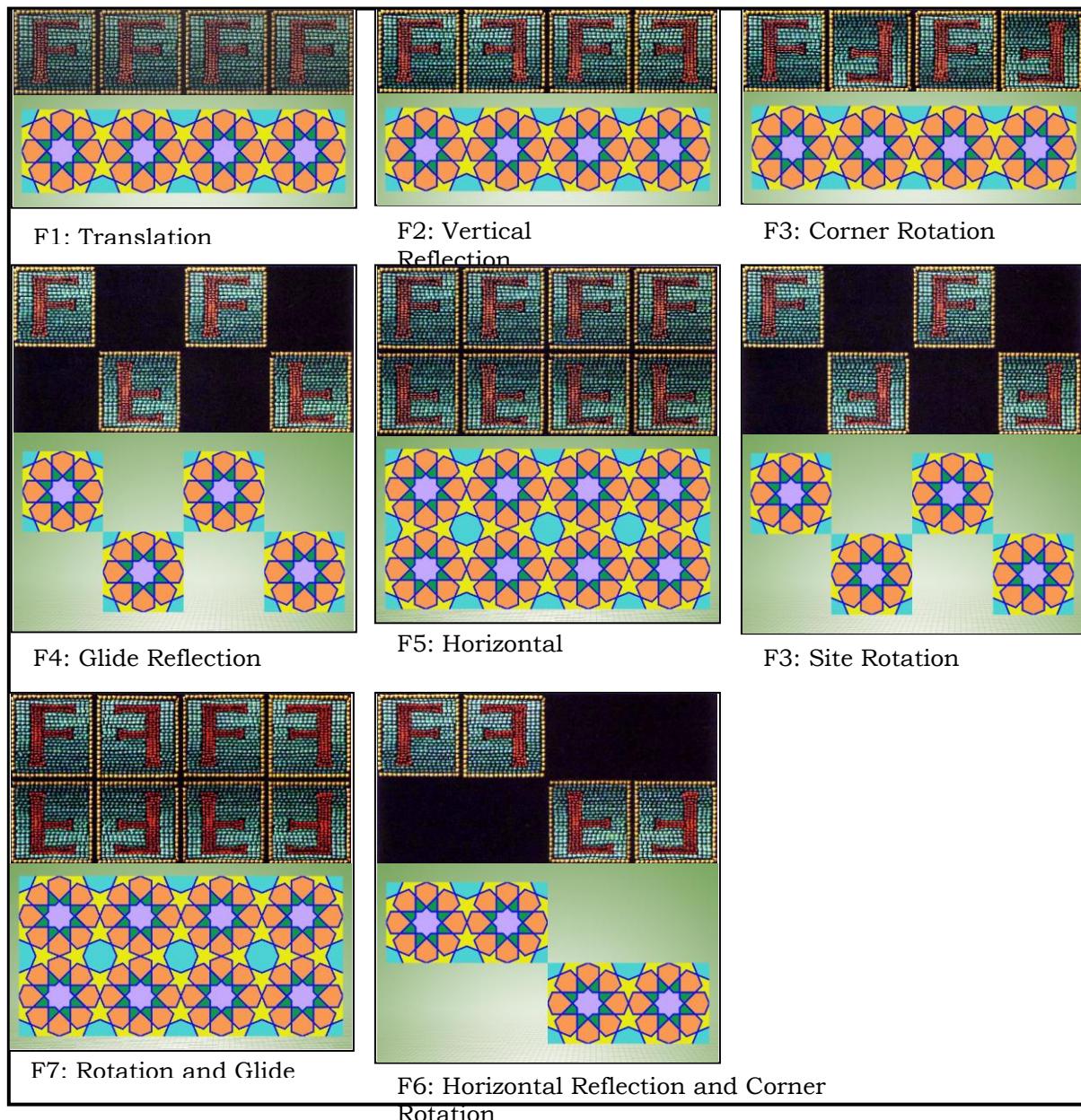


Figure 1: Compression analysis between symmetrical and asymmetrical designs of seven frieze patterns showing the pattern of IGP stays the same with no change.

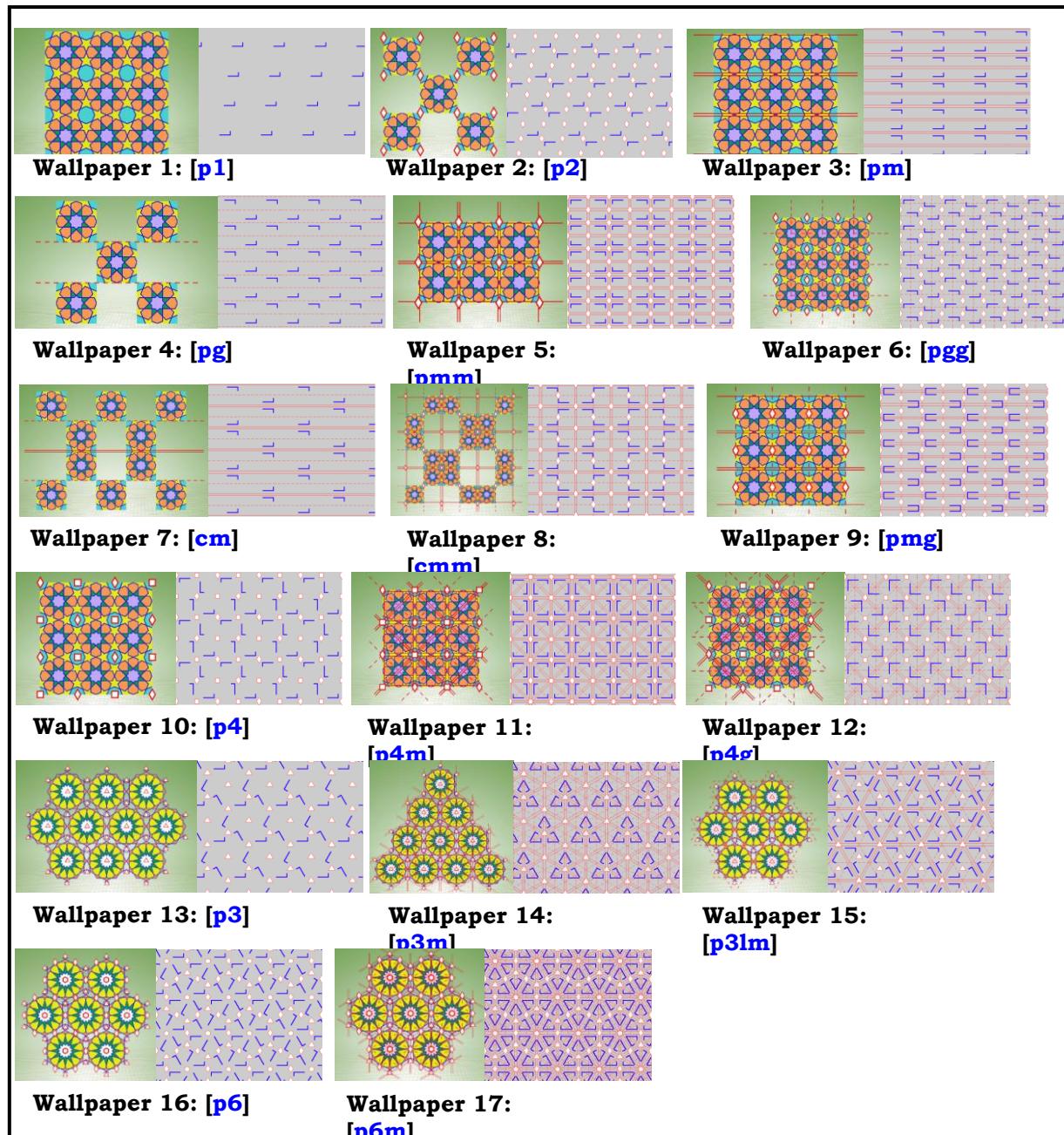


Figure 2: Compression analysis between symmetrical and asymmetrical designs of 17 wallpaper patterns showing the pattern of IGP stays the same with no change.

[Els93] [Sal91] [Cas99] [Els76] [Alj03b] developed different design methodologies to create Islamic geometric patterns. The earlier concepts of design methodologies involve; square grid, intersecting parallel lines, dot, overlapping grid, dual, connecting midpoints of sides, compatible shapes, dissecting shapes, star polygons, ruler/compass and numerical.

In this paper we elaborate on the Normalisation Methodology we developed earlier [Alj03b] which is based on concept of the sub-motif grid and design of the unit pattern. The computer software we developed for the classification of IGP, provides a platform for drawing the sub-motif grid of the pattern.

Once the design in the sub-motif finished with the means of rotation and reflection the software generates the unit pattern design of IGP.

4. AUTOMATIVE CLASSIFICATION AND DESIGN

The objective of this paper is to propose a new classification method and designing tool for developing Islamic geometric pattern using computer software based on the gridding system used to generate a unit pattern. Usually Islamic geometric designs are named and classified on basis of their visual appearance such as pentagonal, hexagonal, heptagonal, octagonal, nonagonal, etc... Such classification is too general and misleading. Therefore, instead of looking at Islamic geometric designs in terms of star/rosettes naming, it is proposed to classify them based on their girding attributes and geometrical properties. According to grid method classification, a deconstruction and normalization is applied to the unit pattern of star/rosette. Normalisation, which is a process of reducing to a norm or standard, can be done by identifying individual geometries and grids that make up the unit designs of Islamic geometric art. Figure 3 and 4 are illustrating the process of division of the circle, drawing the geometry and the gridding system. When grids are finished the classification is named. The process of classification and reconstruction of the gridding system using proposed computer software could be taken up in the following stages:

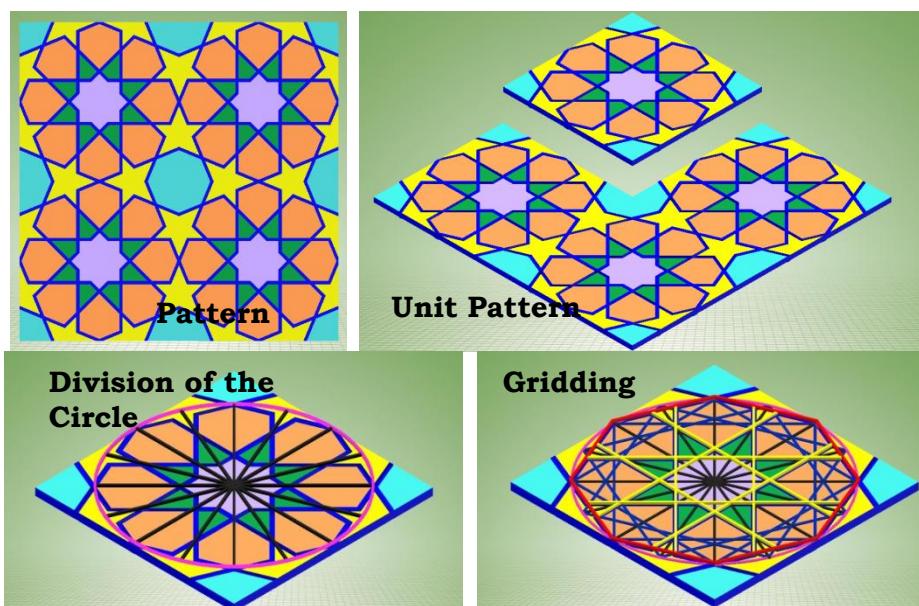


Figure 3: Identifying the pattern, the unit, the division, the geometry and the gridding.

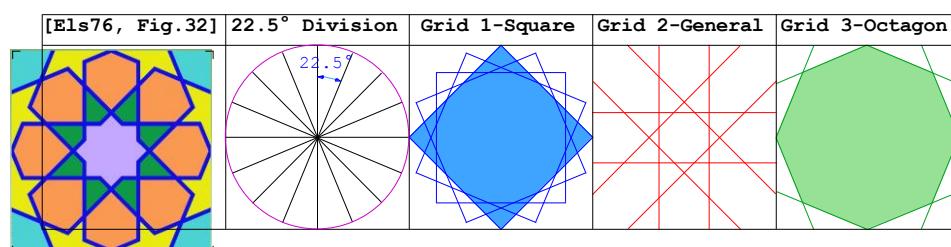


Figure 4: Final classification – 22.5° Triple Grid Quadrilateral (22.5° 3Q)

The Circle Division Selection Stage, The Geometry Grid Selection Stage, The Frame Selection Stage, The General Grid Selection Stage, The Classification Stage, The Grid Extension Stage, The Sub-Motive Selection Stage, The Design Stage, and The Pattern Stage.

The Circle Division Selection Stage: In the classification menu of the software the circle division stage can be initiated by choosing the radius of the circle (R) which restricts the placement of grids within the parameters of the size as shown in figure 5. In the same menu the circle can be divided into (x) number of cones based on the star/rosette design. As in our example the circle has been divided by 16 points in order to obtain the angle of design as 22.5°. Automatically the classification software displays the value of the angle on the screen as in figure 5. The display of the angle on the screen is the 1st part of our naming convention for classification. In the same menu user can select the unit of measurement, pen size and pen colour.

```
g1.DrawEllipse(dr, R);
Double aa = Convert.ToDouble(Input_angle);
Double bb = (360 / Convert.ToDouble(Input_angle));
dr = new Pen(c_color, 1);
Point start = new Point(x + R, y);
for (int i = 0; i < Convert.ToInt32(bb); i++)
{
    g1.DrawLine(dr, center, start);
    double fSine = Math.Sin(aa / (360 / (2 * Math.PI)));
    double fCosine = Math.Cos(aa / (360 / (2 * Math.PI)));
    start = new Point(center.X + Convert.ToInt32((start.X - center.X) * fCosine) -
        (Convert.ToInt32((start.Y - center.Y) * fSine)), center.Y +
        Convert.ToInt32((start.X - center.X) * fSine) + (Convert.ToInt32((start.Y - center.Y) * fCosine)));
    aa = +increment;
}
```

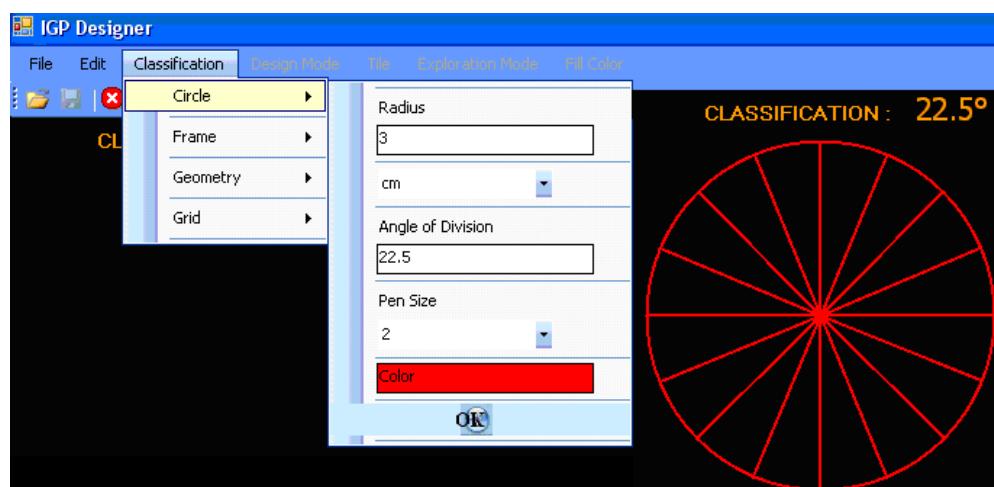


Figure 5: Drawing a circle of given radius and dividing the circle at given angle of division.

and types of geometry (triangular, quadrilateral, pentagonal or heptagonal) to design the gridding system of appropriate star/rosette. In our example we are selecting 4 squares from the quadrilateral menu inscribed in the circle drawn at equidistant points as in figure 6. Also we are selecting an octagon as in figure7. Automatically the classification software displays the type of geometry selected as in figure 7. The display of the geometry on the screen is the 2nd part of our naming classification convention.

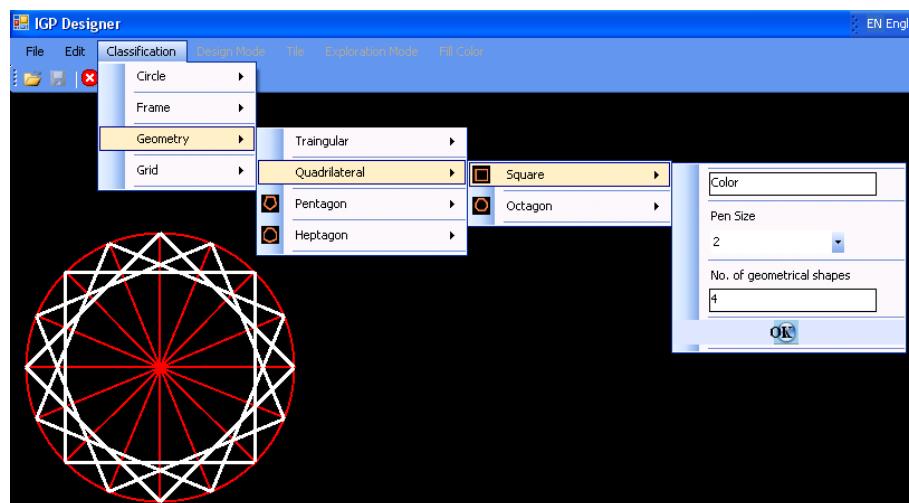


Figure 6: Selecting four squares inscribed in the circle drawn at equidistant points.

In the same menu user can select the pen size and the pen colour. In the naming convention of classification used in software the logic has been adopted to normalise the types of geometries used into their norms. In example, the design requires 4 squares and 1 octagon. When Octagon is normalised it makes 2 squares. Therefore the software classifies the naming convention as quadrilateral. Similarly when triangles or their multiples of geometry are used, the software logic will normalise the geometry into triangular classification.

```
double angle, rotateangle;
{
    angle = 360 / Convert.ToDouble(no_shape) / (360 / (2 * Math.PI));
    rotateangle = angle / 4;
    gangle = (Math.Round((angle * (360 / (2 * Math.PI)) / 4), 2)).ToString();
    if (angle1 == 0) { circle_angle_division.Text = gangle; }
    dr = new Pen(g_color, g_pen);
    for (int i = 0; i < (Convert.ToInt16(no_shape) - 1); i++)
    {
        Point tri4, tri5, tri6, tri7
        double fSine = Math.Sin(rotateangle);
        double fCosine = Math.Cos(rotateangle);
        tri4 = new Point(center.X + Convert.ToInt32((b1.X - center.X) * fCosine) -
        (Convert.ToInt32((b1.Y - center.Y) * fSine)), center.Y + Convert.ToInt32((b1.X -
        center.X) * fSine) + (Convert.ToInt32((b1.Y - center.Y) * fCosine)));
        tri5 = new Point(center.X + Convert.ToInt32((b2.X - center.X) * fCosine) -
        (Convert.ToInt32((b2.Y - center.Y) * fSine)), center.Y + Convert.ToInt32((b2.X -
        center.X) * fSine) + (Convert.ToInt32((b2.Y - center.Y) * fCosine)));
        tri6 = new Point(center.X + Convert.ToInt32((b3.X - center.X) * fCosine) -
        (Convert.ToInt32((b3.Y - center.Y) * fSine)), center.Y + Convert.ToInt32((b3.X -
        center.X) * fSine) + (Convert.ToInt32((b3.Y - center.Y) * fCosine)));
        tri7 = new Point(center.X + Convert.ToInt32((b4.X - center.X) * fCosine) -
        (Convert.ToInt32((b4.Y - center.Y) * fSine)), center.Y + Convert.ToInt32((b4.X -
        center.X) * fSine) + (Convert.ToInt32((b4.Y - center.Y) * fCosine)));
        dr.DrawLine(pen, center, tri4);
        dr.DrawLine(pen, center, tri5);
        dr.DrawLine(pen, center, tri6);
        dr.DrawLine(pen, center, tri7);
        dr.DrawLine(pen, tri4, tri5);
        dr.DrawLine(pen, tri5, tri6);
        dr.DrawLine(pen, tri6, tri7);
        dr.DrawLine(pen, tri7, tri4);
    }
}
```

```

tri5 = new Point(center.X + Convert.ToInt32((b2.X - center.X) * fCosine) -
(Convert.ToInt32((b2.Y - center.Y) * fSine)), center.Y + Convert.ToInt32((b2.X
- center.X) * fSine) + Convert.ToInt32((b2.Y - center.Y) * fCosine));
tri6 = new Point(center.X + Convert.ToInt32((b3.X - center.X) * fCosine) -
(Convert.ToInt32((b3.Y - center.Y) * fSine)), center.Y + Convert.ToInt32((b3.X
- center.X) * fSine) + Convert.ToInt32((b3.Y - center.Y) * fCosine));
tri7 = new Point(center.X + Convert.ToInt32((b4.X - center.X) *

```



Figure 7: Selecting an octagon geometry circumscribing the circle.

```

fCosine) - (Convert.ToInt32((b4.Y - center.Y) * fSine)), center.Y +
Convert.ToInt32((b4.X - center.X) * fSine) + Convert.ToInt32((b4.Y - center.Y)
* fCosine));
g1.DrawLine(dr, tri4, tri5); g1.DrawLine(dr, tri5, tri6);
g1.DrawLine(dr, tri6, tri7);
g1.DrawLine(dr, tri4, tri7);
rotateangle = rotateangle + angle / 4;
}

```

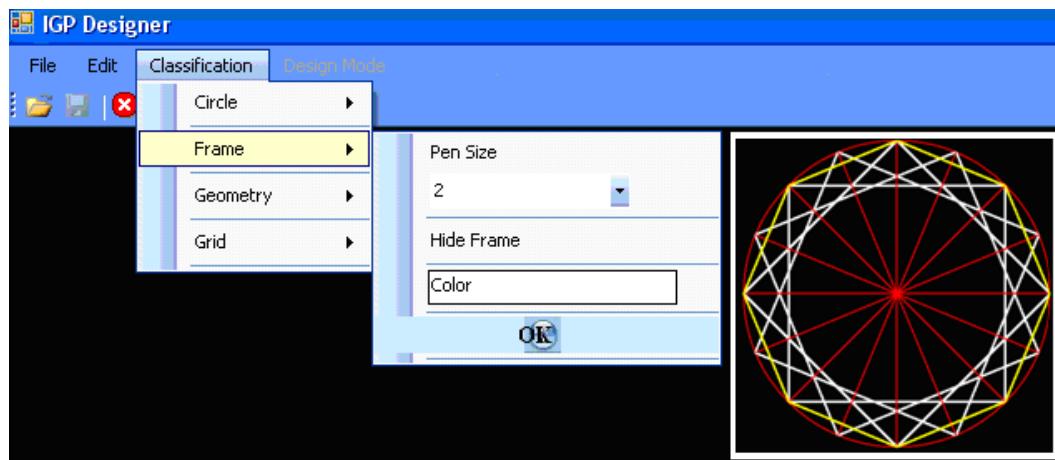
The Frame Selection Stage: In the classification menu user can circumscribe the circle with a frame (external boundary) as in figure 8. External boundaries (usually a square or a rectangle) are regarded as indispensable in some designs. The external boundaries are associated with extended lines of grids and geometries. The external boundaries are phantom to the star/rosette design and its presence cannot be always confirmed until and unless the existence of external mesh can be traced.

```

Point fr1, fr2, fr3, fr4;
fr1 = new Point(x, y);
fr2 = new Point((2 * R) + x, y);
fr4 = new Point(x, (2 * R) + y);
fr3 = new Point((2 * R) + x, (2 * R) + y);
g1.DrawLine(dr, fr1, fr2);
g1.DrawLine(dr, fr2, fr3);
g1.DrawLine(dr, fr3, fr4);

```

g1.DrawLine(dr, fr4, fr1);



The General Grid Selection: In the classification menu user can build array of grids as in figure 9. After finishing each grid user must click Finish in order for the software to store a count and details of total grids. When the gridding stage is finalised, by clicking the button Classify the software will classify the grid design accordingly. The displacement of number of grids on the screen is the 3rd part of our naming classification convention. In the same menu user can select the pen size and the pen colour. In addition to the grids, in some unit-pattern the grid lines are needed to be extended to accommodate the design.

The Grid Extension Stage: The extension feature has been added to the software where the required grid lines extended to border the notational boundary (the frame). The extended lines of grids and geometries would generate intersection points to accomplish the seamless mesh in the external zone within the notational boundary as in figure 9.

The Classification Stage: This is the final stage of classification software where the structural system of the star/rosette is ready for design. The classification process is completed and is shown on the screen as in figure 10.

22.5° Triple Grid Quadrilateral (22.5° 3Q) (22.5°) represents the angle of sub-motif. (3) represents the number of grids used for the design development and (Q) denotes the geometrical type, i.e., Quadrilateral.

The Sub-Motif Selection Stage, The Design Stage, The Pattern Stage: The sub-motif selection stage initiates the designing process and finally the pattern development. The core objective of this stage is to select the sub-motif. And this is done automatically by the software by recognising the angle of division of the circle as the angle of the design and cuts the design grid view into the Sub-Motif. The angle can be altered to Motif or even Full-Patter as per the user. In the Sub-Motif designing stage the user has the choice of either Repeat & Rotate or Mirroring & Rotate and other parameters

as in figure 11. Finally the user can develop the Full Pattern in x-y direction as in figure 12.

6. CONCLUSION

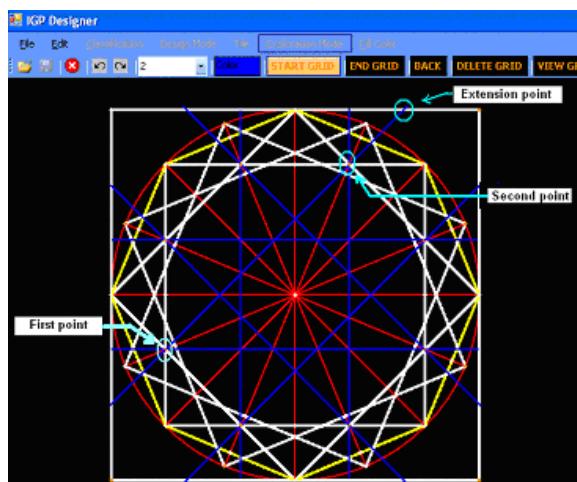


Figure 9: Extending the necessary

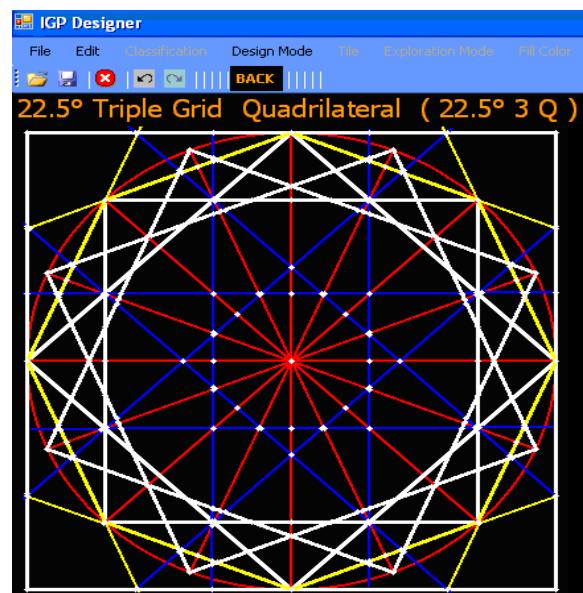


Figure 10: The displacement of classification on the screen with complete grids, geometries, extensions and the frame.

Islamic geometric patterns have long been classified based on pattern theories. We found that these pattern theories are not accurate enough to classify the IGP based on its unit pattern, but are classified on basis of their arrangements and not the designs. IGP which is symmetric in nature does not show any effect when arranged according to pattern theories. This paper stated the problem with a clear objective of classifying the IGP on the basis of their gridding and geometric attributes which will help us to classify any star/rosette. The classification gives the reader information on the angle of design, on the type of geometry and number of grids used in achieving the design of IGP. On the design part we found that IGP have led to several significant design methodologies that advocate the Gridding, Designing and Exploration in different ways. The fundamental principles of all these design methodologies vary based on the experiences and disposition of their proponents. We have developed a new design methodology in conjunction with classification using the grid methodology which can be used to design IGPs based on the sub-motif grid. The researcher developed an ambitious computer software program using C Sharp in .NET platform. The software enable its users to create IGP from the scratch in stages and arrive at its classification with ease. The program will be an excellent tool in analysing IGP for designers, architects, mathematicians and academic fraternity working in related areas.

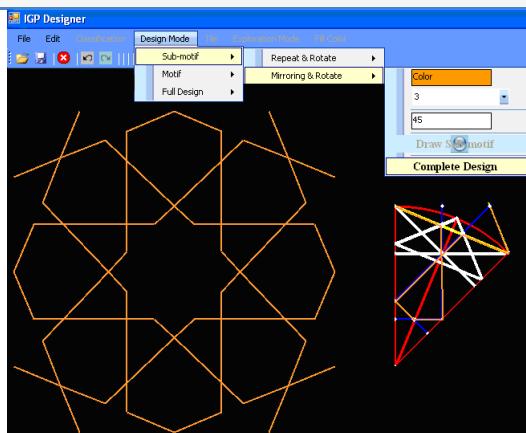


Figure 11: Once the design is drawn over the motif or sub-motif, click on complete Design button and design only will be displayed and it will be mirrored and rotated to complete the circle.

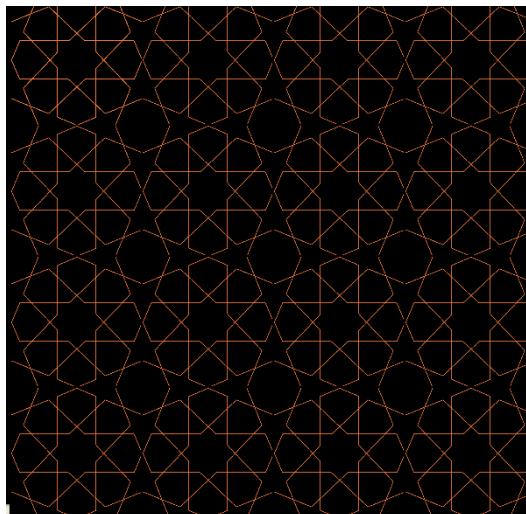


Figure 12: The design developed is tiled in x and y direction as per the inputs.

7. REFERENCES

1. [Gla99] Glassner Andrew: Andrew Glassner's Notebook: Recreational Computer Graphics Morgan Kaufmann Publishers, San Francisco, CA. U.S.A. (1999)
2. [Lee98] Lee X.: The 17 Wallpaper Groups: (1998)
http://www.xahlee.org/Wallpaper_dir/c5_17WallpaperGroups.html
3. [Gru86] Grunbaum Branko, Zdenka Grunbaum and Shephard G. C.: Symmetry in Moorish and other Ornaments: Comp. and Maths, with Appls, Vol. 12B. Nos. 3/4, p. 641-653. Pergamon Press Ltd. (1986)
4. [Sch89] Schattschneider Doris: The Plane Symmetry Groups: Their Recognition and Notation Computer & Mathematics with Applications, Vol. 17, No. 1-3, pp. 13-15, Pergamon Press Ltd, U.K.

(1989)

5. [Mul44] Muller E. Gruppentheoretische and Strukturanalytische: Untersuchung der Maurischen Ornamente aus der Alhambra in Granada: PhD Thesis: University of Zurich. Baublatt, Ruschlikon (1944)
6. [Bel56] Belov N. V.: Moorish Patterns of the Middle Ages and the Symmetry Groups: [In Russian] Kristallografya 1.47 476 (1956), English Translation; Soviet Physics Crystallography 1.482 483 (1956)
7. [Cri76] Critchlow K.: Islamic Pattern: An Analytical and Cosmological Approach: (1976) by Thames & Hudson Ltd, ISBN 0-500-27071-6
8. [Els93] El-Said Issam: Islamic Art and Architecture: The System of Geometric Design: Grant Publishing Limited, U.K. (1993): ISBN 1-873938-45-5
9. [Sal91] Salman Amer Shaker: Computer Graphics Studies of Islamic Geometrical Patterns and Designs, Ph.D. Thesis: University of Wales, Cardiff, U.K. (1991)
- 10.[Cas99] Castera Jean-Marc: Arabesques, Decorative Art in Morocco: ACR Edition International, Courtevoie Paris (1999)
- 11.[Alj03b] Aljamali A. and Banissi E.: Normalisation and Exploration Design Method of Islamic Geometric Patterns: London, U.K.: Published by the IEEE Computer Society, pages: 42-48 (2003): ISBN 0-7695-1985-7
- 12.[Cay06] Cayley Arthur: Mathematician Laureate of the Victorian Age (Hardcover) by Tony Crilly 2006 The Johns Hopkins University Press ISBN 0-8018-8011-4
- 13.[Syl08] Sylvester James: The Collected Mathematical Papers of James Joseph Sylvester, Volume 2 (Ams Chelsea Publishing) by James Joseph Sylvester (Hardcover - Nov 21, 2008) ISBN: 978-0-8218-4725-1